

# ECON 2201 Supply & Demand Review

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*N.B.: I've changed the example slightly from that used in discussion. Try to solve for equilibrium and compute elasticities on your own first, then continue reading to check your results.*

Suppose the market for widgets in a given month is described by the following system of equations:

$$q_d = 20 - 2p \tag{1}$$

$$q_s = -4 + 2p \tag{2}$$

where equation 1 is the demand function and equation 2 is the supply function. The slope of the demand function is negative, which corresponds to the “law of demand”: other things equal, as price increases, buyers demand a lower quantity. (Of course, the demand for widgets is a function of many things, including incomes and prices of other goods, not just the price of widgets. But we’re holding those constant here, so their effect is collapsed into the constant term.)

Exactly how does the quantity demanded change as price changes? The demand function’s slope of  $-2$  gives us one way to answer this question. But notice that if we were to change the units (say, to dozens of widgets or tons of widgets, or to euros instead of dollars), the slope would change. This is the problem that Alfred Marshall’s concept of *elasticity* solves. By relating the percentage change in quantity demanded to the percentage change in price, we obtain a measure that stays the same even when we change our units of measurement.

As we discussed in class, a linear demand function like that above does not have the same elasticity at every price<sup>1</sup>. How do we find the price

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<sup>1</sup>What kind of demand function would have a constant elasticity? One of the general form  $q_d = ap^{-\epsilon}$ , where  $a$  is some constant, would have elasticity  $-\epsilon$  at every price.

elasticity of demand at any given price?

Recall that:

$$\epsilon = \frac{\% \Delta q_d}{\% \Delta p} = \frac{\frac{\Delta q_d}{q_d}}{\frac{\Delta p}{p}} = \frac{\Delta q_d}{\Delta p} \frac{p}{q}$$

But  $\frac{\Delta q_d}{\Delta p}$  is just the slope of the demand function<sup>2</sup>, and the slope is  $-2$  at all feasible prices, so to find elasticity of demand at any price we can simply use

$$\epsilon = -2 \frac{p}{q_d}$$

So, for example, at a price of 5, we have  $p = 5$ ,  $q_d = 20 - 2(5) = 10$ , and therefore  $\epsilon = -2 \frac{5}{10} = -1$ .

Until now we've been talking about the slope of the demand function in  $(p, q)$  space. But remember that, following Marshall, we graph the "demand curve" in  $(q, p)$  space. Thus, to graph the demand curve, we first need to find the *inverse demand function*. To do that, we simply solve equation 1 for price instead of quantity. (Similarly, we solve for  $p$  in equation 2 to get the supply curve.) Thus, we graph:

$$p = 10 - \frac{1}{2}q_d \tag{3}$$

$$p = 2 + \frac{1}{2}q_s \tag{4}$$

to get our usual demand and supply curves. Note that, for example, the slope of the inverse demand function, i.e. the slope of the demand curve, is  $-\frac{1}{2}$ , not  $-2$ . Confusing the two is a common mistake, so be careful to use the correct one when calculating elasticities. (When in doubt, start with our definition of elasticity as  $\frac{\% \Delta q_d}{\% \Delta p}$  and work it out from there.)

Finally, we can find the equilibrium price and quantity by solving the system in equations 1 and 2. Although there are three unknowns and only two equations, recall that in equilibrium  $q_d = q_s$ . Thus, we can set  $q_d = q_s$ , or  $20 - 2p = -4 + 2p$ , and solve to get  $p = 6$  as our equilibrium price. Then we can substitute this value into either equation to get  $q = 8$  as the corresponding equilibrium quantity. (It's a good idea to substitute into *both* equations, actually. If you don't get the same answer, you made an algebra mistake!) If we graph the supply and demand curves, this is the point at which they intersect.

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<sup>2</sup>Or, in calculus terms,  $\frac{\delta q}{\delta p}$ .